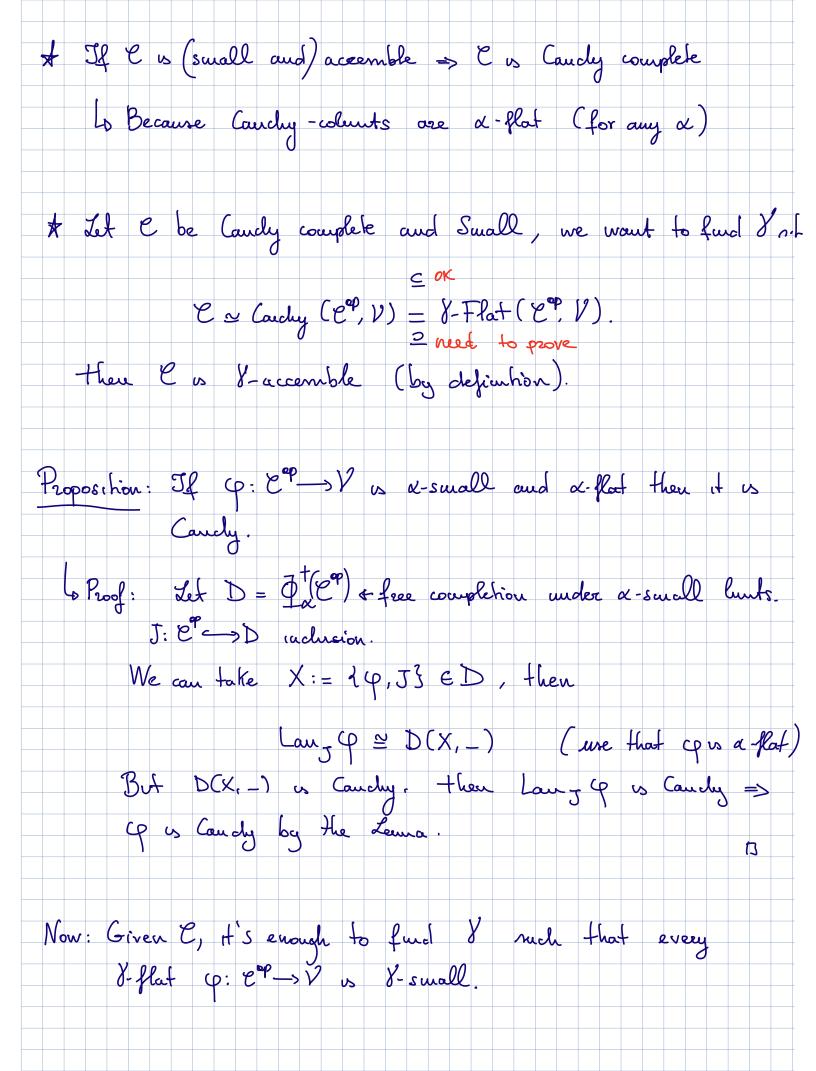
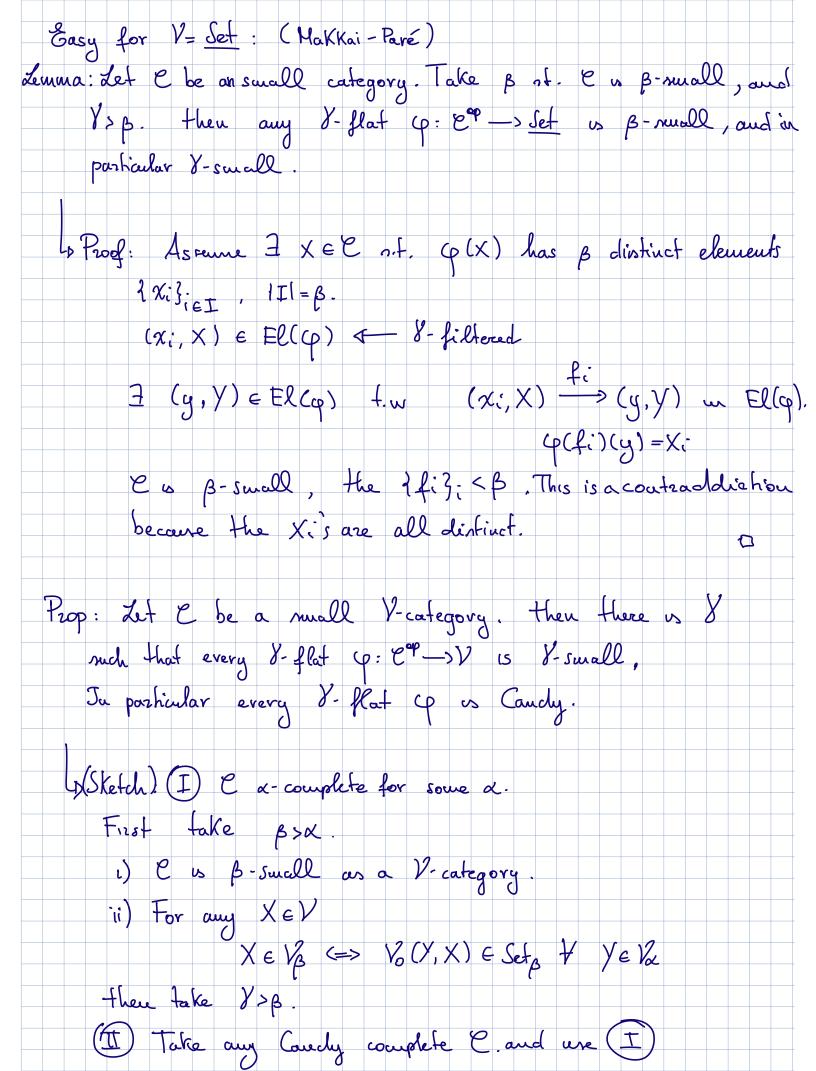


L'From now on every cardinal as 200. Def: Let A be a V-category. . A is conically accemble if it is the free cocoupletion of a small V-category under x-fildered colimits, for some «. • A accemble if A ~ x - Flat (C,V), for a mall C and some d. We work with this. A small V-category is conically accemble (=> it leas splittings of idempokents. III) Flatuess Def: Let  $\varphi: \mathcal{C}^{op} \longrightarrow Y$  be a V-functor, we say that  $\varphi$  is (Kelly). &-small if i)  $\mathcal{C}$  us  $\alpha$ -small:  $\times$   $O(\mathcal{C}) < \alpha$ ,  $\mathcal{C}(X,Y) \in V_{\alpha}$  $\ddot{u} \quad \varphi(X) \in \mathcal{V}_{\mathcal{X}} \quad \forall X \in \mathcal{C}.$ • &-mall limits are those weighted by x-mall functors. Def: We say that cp: e<sup>op</sup> ->V is a plat if y 1 Law y  $e^{\circ r} \rightarrow V$ Lange is a - continuous.

But Lawy q ≥ q × \_. Equivalently: q coluents commute in V with x-small limits. Prop: Let  $cp: \mathcal{C}^{op} \longrightarrow V$  be a  $\mathcal{V}$ -functor; TFAE: i) q vs x-glat ii)  $\varphi \star - \varphi$  preserves  $\alpha$ -mall limits of representables  $T_4 \ e^{\mathbf{ap}}$  is  $\alpha$ -complete, moreover: iii) (p 15 x- coatimous. Lemma: Green q: esp-V and J: e-D. c) cp is a flat => Langop cp is ac-flat. ic) Jos f.f and langop(p is a - flat => cp is a - flat as well.  $\frac{Def}{I.e.} : (e,V) \longrightarrow V \text{ is Cauchy if it is a flat tax.}$   $i.e. \quad (e,V) \longrightarrow V \text{ is continuous.}$ (=> q-colicients are preserved by any V-functor. (p- colimts are absolute). We say that C is Cauchy complete if it has all absolute coliunts, i.e all coliunts weighted by Cauchy fantors. Fact: C is Cauchy complete (=> C ~ Cauchy (C, V) = [C, V] IV) The Result.





theorem : A small V-category & is accemble <> it is Cauchy Couplete. Prop: Let K be locally &-presentable. Then there exist I much that any &-accessible (preserves &-filt colimnts) V-functor  $F: K \longrightarrow \mathcal{L}$  ( $\mathcal{Z}$  a-accemble V-category) is continuous 't and only if it preserves 8-mall limits. 4) Proof: Take Y as in the Proposition for C=Kx. Mog L = V.• F & continuous • F5 × - prenzves &-small I lunts of representables · FJ v 8-flat • FJ is Cauchy · FJX\_ is couhimous\_ F is continious. Corollary: XEV is dualizable (=> XO\_ is V-continuous.