A higher-dimensional Eckmann–Hilton argument

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Abstract. (Joint work with **Eugenia Cheng, School of the Art Institute of Chicago**)

The Eckmann–Hilton argument plays a subtle and crucial role behind the scenes in higher-dimensional category theory. It can show us where hidden commutativities arise from cells with identity boundaries being able to commute past each other, given enough dimensions. The basic case tells us that doubly-degenerate 2-categories give commutative monoids; more precisely, a doubly-degenerate 2-category "is" a set with two multiplications on it satisfying interchange, and the Eckmann–Hilton argument tells us that those multiplications must be the same and commutative [3].

Given more dimensions, more nuance is possible. It is considered well-known that doubly-degenerate weak 3-categories "are" categories with two weak monoidal structures satisfying weak interchange, and that a weak Eckmann–Hilton argument shows that this amounts to a braided monoidal category [4]. However, the classic proofs of this do not explicitly provide the details of the generalisation of the Eckmann–Hilton argument [5], [1].

In this talk I will quickly explain why we are interested in digging into the details of a well-known result, while also trying to highlight the curious combinatorial elements of the higher-dimensional Eckmann–Hilton arguments. All of this being to further our work with semi-strict higher categories, to enable us to remove the quotation marks around "is" in the statement

An n-degenerate $(n+1)$-category "is" a symmetric monoidal category

to give a satisfying, fully algebraic proof of one part of the stabilisation hypothesis [2].

References

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